COMPLEX CONDUCTIVITY AND SURFACE IMPEDANCE OF SUPERCONDUCTOR BASED ON MODIFIED TWO FLUID MODEL

KENJI OHSHIMA

ABSTRACT

Complex conductivity of the superconductor (CCS) is a principal parameter in the case of estimation for surface impedance of the superconductor. Conventional investigations on CCS are based on the two fluid model. But it does not agree with experimental result especially at the low temperature. We propose then a modified two fluid model for the superconductor provided that moving electron pairs have the characteristic of equivalent loss which depends on the RF field, and both densities of super and normal electron are related by the Rate equation. The CCS and surface impedance for superconductor are formulated by using a modified two fluid model. Calculated CCS and surface impedance agree with measured results.

1. INTRODUCTION

The high frequency properties of the superconductor depend on the surface impedance [1] . For the purpose of estimation of the surface impedance of a super conductor, it is necessary to formulate the complex conductivity of super conductor (CCS). Conventional investigation on CCS is based on the two fluid model. In the two fluid model, it is assumed that electron pair do not collide with the phonon [1]. However, the real part of CCS and surface impedance that derived from two fluid modeldo not agree with measured results [2][3]. Accordingly, various fluid models have been proposed in order to solve above problem [4][5]. But a sufficient model have not found yet.

We propose a novel modified two fluid model for the superconductor assuming that moving electron pair have the equivalent loss depending on the RF field, and both densities of super and normal electron are related by the Rate equation[6]. That is, by using this model, we formulate the equation of motion on the electron pair and normal electron, and then constitute an equation of the super and normal current densities. Modified CCS are formulated from the derived equation of current density. The surface impedance for superconductor is also determined from modified CCS. Finally we show that the numerical results of the real part of modified CCS and surface impedance obtained by this model agree with measured results.

2. ANALYSIS

The conditions to derive the modified equation of motion (MEM) for the charged fluid of super and normal electron particles, and the equation of super and normal current density (ECD) are as follows:

[I] Moving electron pair in the superconductor have the equivalent loss in RF, so that the equation of motion for electron pair may be introduced by equivalent relaxation time [4].

[II] The densities of super and normal electron are depended on temperature, so that the both electron densities may be related by Rate equation[5].

When condition [I] hold, the MEM for electron pair and normal electrons becomes the same equation of [4]. We now define super current density as $\mathbf{J}_s = n_s q \mathbf{v}_s$ and normal current density as $\mathbf{J}_n = n_n q \mathbf{v}_n$, where n_s is the super electron particle density, n_n is the normal electron particle density, $\mathbf{V}_s$ is the super electronparticle velocity, $\mathbf{V}_n$ is the normal electron particle velocity, and q is the electron charge. Using , $\mathbf{J}_s$, $\mathbf{J}_n$ and MEM, we obtain following ECD:

$$\frac{d\mathbf{J}_s}{dt} + \left[\frac{1}{\tau_s} - \frac{1}{n_s}\cdot\frac{dn_s}{dt}\right]\mathbf{J}_s = \frac{q^2 n_s}{m}\mathbf{E} \qquad (1),$$

$$\frac{d\mathbf{J}_n}{dt} + \left[\frac{1}{\tau_n} - \frac{1}{n_n}\cdot\frac{dn_n}{dt}\right]\mathbf{J}_n = \frac{q^2 n_s}{m}\mathbf{E} \qquad (2),$$

where m is mass of an electron particle, τ_s is the equivalent relaxation time depended on the moment of inertia of super electron in RF field, τ_n is the normal electron relaxation time, and $\mathbf{E}$ is the electric field. From condition [II], the Rate equation [5] between dn_s/dt in (1) and dn_n/dt in (2), and the relaxation time τ_h depending on thermal interaction are defined as follows:

$$\frac{dn_n}{dt} = -\frac{dn_s}{dt} \qquad (3),$$

$$-\frac{1}{n_n}\cdot\frac{dn_n}{dt} = \frac{1}{\tau_h} \qquad (4).$$

From the ECD of (1) and (2), we find that $\mathbf{J}_s$ and $\mathbf{J}_n$ may be expressed as follows:

$$\mathbf{J}_s = \frac{q^2 n_s}{m}\cdot\frac{\tau_{ss}}{1+j\omega\tau_{ss}}\mathbf{E} \qquad (5),$$

$$\mathbf{J}_n = \frac{q^2 n_n}{m}\cdot\frac{\tau_{nn}}{1+j\omega\tau_{nn}}\mathbf{E} \qquad (6),$$

hence, τ_{ss} in (5) and τ_{nn} in (6) are represented as follows,

$$\frac{1}{\tau_{ss}} = \frac{1}{\tau_s}\left[1 - \frac{\tau_s}{\tau_h}\cdot\frac{\theta^4}{1-\theta^4}\right] \qquad (7),$$

$$\frac{1}{\tau_{nn}} = \frac{1}{\tau_n}\left[1+\frac{\tau_n}{\tau_h}\right] \quad (8),$$

where, ω is angular frequency, θ is relative temperature ($\theta = T/T_c$, T : temperature, T_c : critical temperature).

Furthermore, the total current density $\mathbf{J}$, CCS (σ_s) and $\mathbf{E}$ are related as follows:

$$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_n = \sigma_s \mathbf{E} \quad (9).$$

Substituting for $\mathbf{J}_s$,$\mathbf{J}_n$ from (5) and (6) into (9),we finally arrive at the following expression for σ_s :

$$\sigma_s = \sigma_{1s} - j\sigma_{2s} = (\sigma_{1s,n}+\sigma_{1s,s}) - j(\sigma_{2s,n}+\sigma_{2s,s}) \quad (10),$$

$$\sigma_{1s,n} = \frac{\sigma_n \theta^4}{1+(\omega\tau_{nn})^2} \quad (11),$$

$$\sigma_{1s,s} = \frac{\tau_{ss}}{\mu_0 \lambda_L^{\,2}\left\{1+(\omega\tau_{ss})^2\right\}} \quad (12),$$

$$\sigma_{2s,n} = \sigma_{1s,n}\omega\tau_{nn} \quad (13),$$

$$\sigma_{2s,s} = \sigma_{1s,s}\omega\tau_{ss} \quad (14),$$

where σ_n is normal conductivity, λ_L is magnetic field penetration depth, μ_0 is permeability [1]. Utilizing the above σ_s, we can show that the surface impedance Z_s of a superconductor is

$$Z_s = \sqrt{\frac{j\omega\mu_0}{\sigma_s}} = R_s + jX_s \quad (15),$$

where R_s is surface resistance, X_s is surface reactance.

3. RESULTS AND DISCUSSION

We will show the numerical results for the real part σ_{1s} of σ_s which contain the parameters with τ_s ,τ_n and τ_h, and then show the surface impedance Z_s by using σ_s. We must now estimate the value of the relaxation times τ_s ,τ_h and τ_n that is used for the estimation of σ_s. First, the value of τ_s is determined by $\tau_s = (4+80/T)/\omega$ which is obtained from curve fitting by the experiment [4]. Next, the value of τ_h must be estimated to the vicinity of τ_s but short than τ_s, because τ_h is caused by thermal interaction of both super and normal electron particles at below T_c. Then, the value of τ_n must be estimated to very shorter than τ_s and τ_h, because τ_n is the relaxation time that is caused by the collision with phonon and normal electron particle. From the above considerations, the values of relaxation times τ_s ,τ_h and τ_n are estimated to be chosen so that $\tau_h \geq \tau_s \gg \tau_n$. The numerical values for τ_s ,τ_h and τ_n are shown in the figures, where τ_h and τ_n are assumed values. Also, the values of σ_n and penetration depth λ_{L0} of London [1] are given together with in the figures. In (10), σ_{1s} are given with $\sigma_{1s,n}$ and $\sigma_{1s,s}$. We now explain physical meaning of $\sigma_{1s,n}$ and $\sigma_{1s,s}$ in Fig.1.

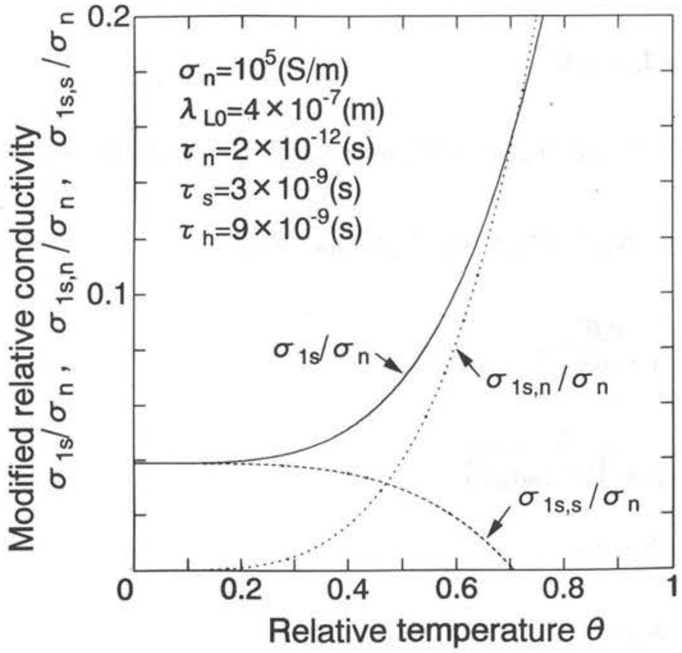

Fig.1 Temperature dependence of modified conductivity.

In Fig.1, it is found that $\sigma_{1s,n}$ depend on $\tau_{nn}(\tau_h ,\tau_n)$ as (11) is similar to the curve which derived from the conventional two fluid model. In the mean while, $\sigma_{1s,s}$ depend on $\tau_{ss}(\tau_s ,\tau_h)$ as (12) come to the finite value according as lowering of θ . Consequently, $\sigma_{1s,n}$ indicate the increment of conductivity is caused by thermal interaction of both super and normal electron particles and collision with phonon and normal electron particle, and then $\sigma_{1s,s}$ is a primary factor in property of the residual resistance that is caused by the equivalent loss in RF of super electron particle and thermal interaction of both super and normal electron particle.

In Fig.2 we show the calculated value and the measured value ($\mathbf{Bi_2Sr_2CaCu_2O_8}, T_c = \mathbf{92K}$:[2]) of σ_{1s}. The measured value of σ_{1s} has the property that comes to the finite value in the range of low temperature. The property come to finite value in the low temperature range, that indicate the reason for being of residual resistance. The calculated value of σ_{1s} that considered $\sigma_{1s,n}$ and $\sigma_{1s,s}$ agree with the measured value. Consequently, σ_{1s} given from analysis can be explain the property of the residual resistance. Furthermore, the calculated value of σ_{1s} in the range of high temperature at below T_c disagree with the measured value [2]. The reason of the disagreement is due to the coherence peak of conductivity in quantum mechanics (microscopic theory).

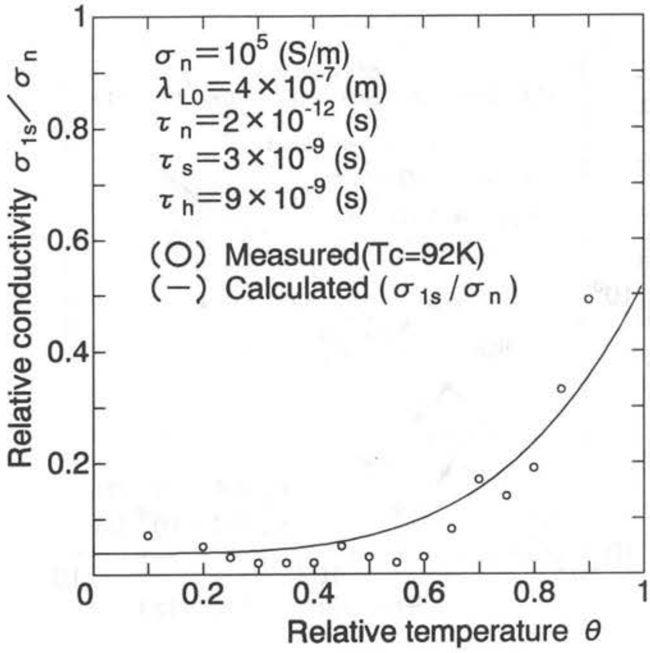

Fig. 2 Temperature dependence of conductivity.

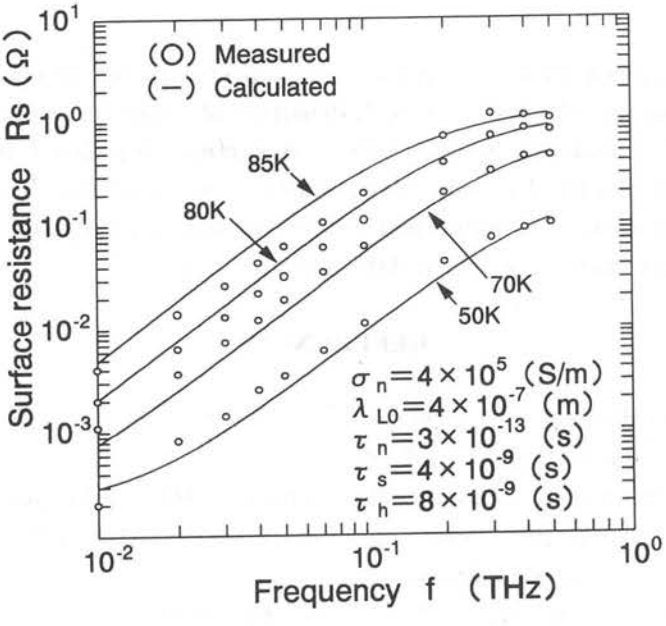

Fig. 3 Surface resistance.

In Fig.3 and Fig.4 we show frequency dependence of the calculated and measured R_s and X_s ($\mathbf{YBa_2Cu_3O_7}$, T_c = **92K** : [3]) respectively. In Fig.3, the calculated R_s that obtained by using σ_s is

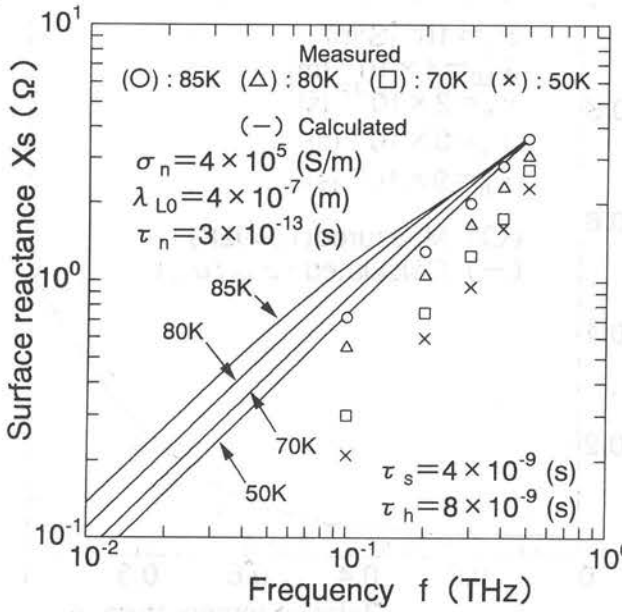

Fig. 4 Surface reactance.

good agreement of measured value. In Fig.4, the calculated X_s that obtained by using σ_s agree with a tendency of measured values.

4. CONCLUSION

We proposed a novel modified two fluid model for superconductor provided that moving electron pair has the equivalent loss in RF, and that both densities of super and normal electrons are related by the Rate equation. The complex conductivity and surface impedance for super conductor were derived from the modified two fluid model. The modified two fluid model can evaluate the residual resistance for a superconductor. The numerical results for conductivity and surface impedance agree with measured results, and consequently validate our analysis.

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